Brunnian Ti-Links

A modular system for creating finite and infinite Brunnian Links

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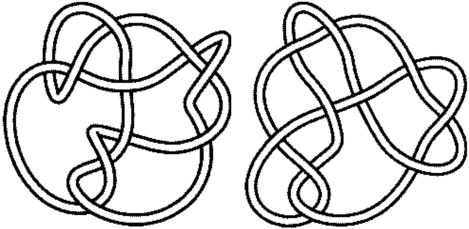
Knot Theory

Two of the main goals are identifying a given knot, and tabulating knots in general. This is hard.



Are these two the same knot?

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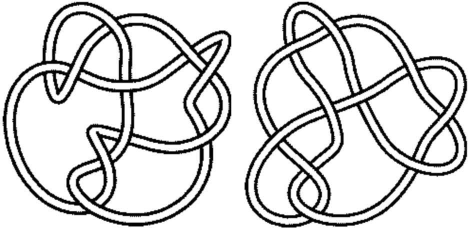


Are these two the same knot?

Now consider that there are 294,130,458 “prime” knots with 19 crossings (OEIS sequence A002863)!

How can we tell them apart?

Invariants

Invariants are quantities that are the same for all diagrams of a knot. Therefore, if two diagrams have different invariants, they must represent different knots.

True/False	Tricolorability, Invertibility, Chirality ...
Number	Crossing #, Unknotting #, Stick # ...
Polynomial	Jones, Alexander, Conway, Kauffman ...
Group(s)	Fundamental Group of the complement, Khovanov Homology ...

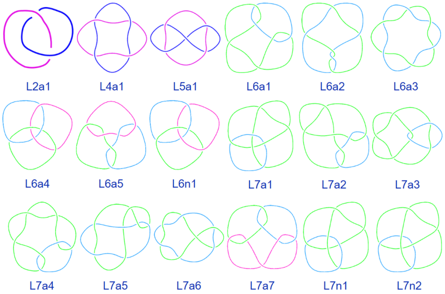
There are a lot of them, in general the more complex quantities differentiate more knots, but there is not a strict hierarchy.

Applications

- Biology - To scale, fitting DNA in human cells is like coiling **200 km of fishing line in a basketball**. As one might expect, this causes a lot of tangling. **Topoisomerases** are enzymes that perform local operations to help in transcription. These allow the knotted DNA of bacteria to separate into daughter cells in bacteria. Quinolones, some of the most common **antibiotics**, work by inhibiting these enzymes, preventing reproduction. This same inhibition is commonly used in **chemotherapy** to stop the division of cancer cells.
- Math - Any closed, orientable, connected **3-manifold** can be obtained by Dehn surgery on a link in S^3 [Theorem by Lickorish and Wallace, 1960].

Brunnian Links

A Link is essentially a knot tied with multiple strings.

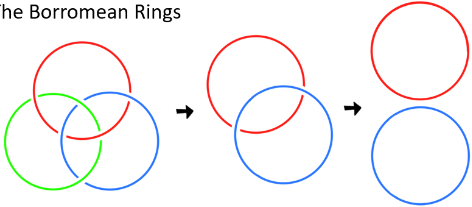


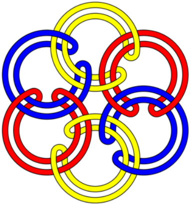
A link is "Brunnian" if it satisfies two properties:

1. It is nontrivial.
2. Every proper sublink is trivial.

Previous constructions

The Borromean Rings





“Rubberband” construction
(infinite family)

Square knot construction
(infinite family)

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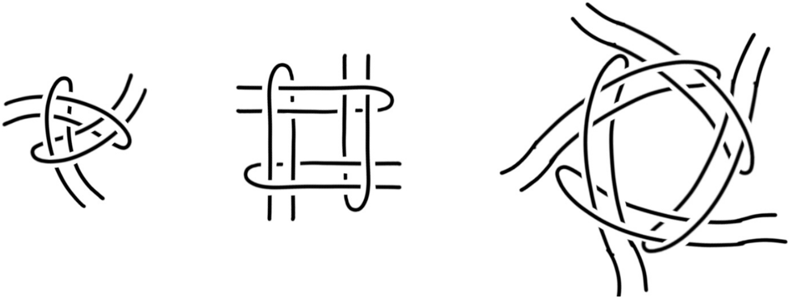
Modules

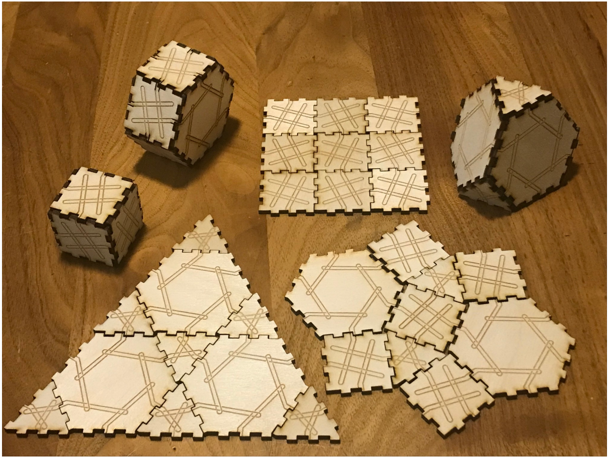
A module with k bands is made by passing each through the next in turn:



...

This is similar to "windmill weaving" strips, some logos of interdependence, a bar trick for balancing utensils on each other, etc.

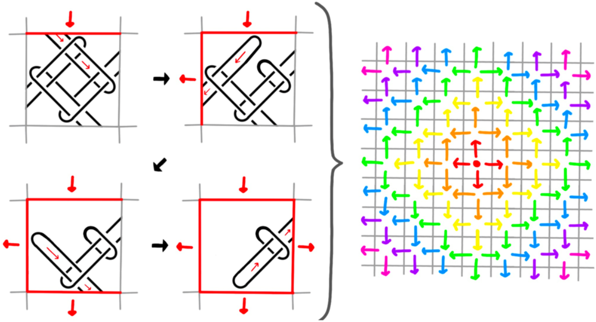
Assembly



This gives a Brunnian link for any polyhedron or infinite tiling! (possibly with some stretching)

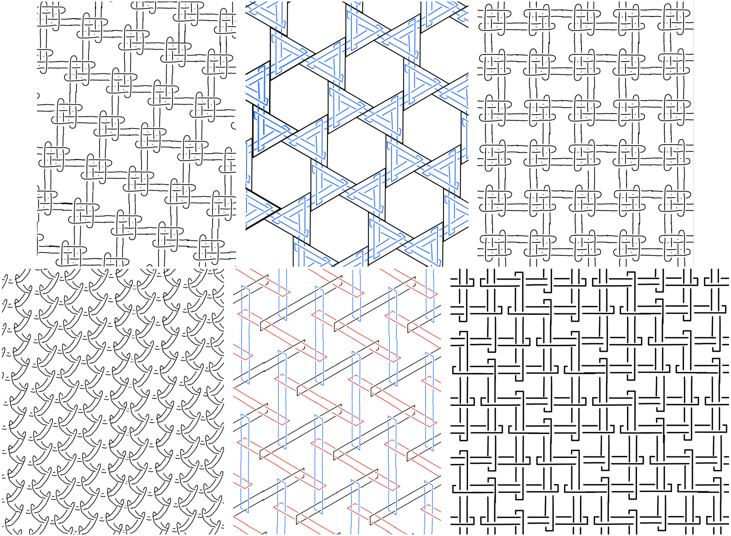
Brunnianity

Freeing one band in a module frees all the others, which propagates through all connected modules.



Showing that these are nontrivial is, unfortunately, not trivial. The associated short paper explain this using a condition from a recent paper.

Examples

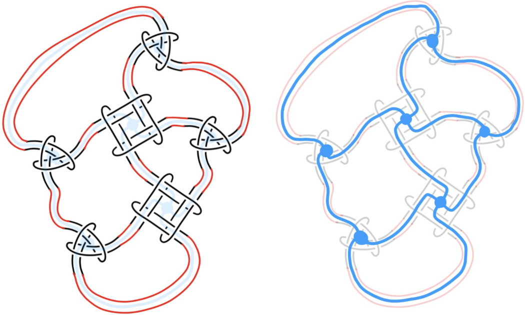


Temari

The polyhedral links are well suited to representation with temari balls:

Graphs

This system is actually more general, each module with k bands corresponds to a vertex of degree k.



Consequently, any graph can be converted into a Brunnian link with this approach, even non-planar ones like $K_{3,3}$ and K_5!

Family Day Activity

I'm running a station making these structures with hair ties on Family Day, Sunday the 4th from 1:00-5:00pm.

Come on by if you'd like to play around with them, the laser cut tiles, or the temari!

Conclusion

Summary:

- We present a system of making Brunnian links corresponding to any graph, as well as various media illustrating this.

Next Steps:

- Find infinite planar tilings with all 17 possible plane symmetries.
- Weaker Brunnianity (removing any k makes trivial, no $i < k$ make nontrivial).

Link to my website, the Outreach page has more details.