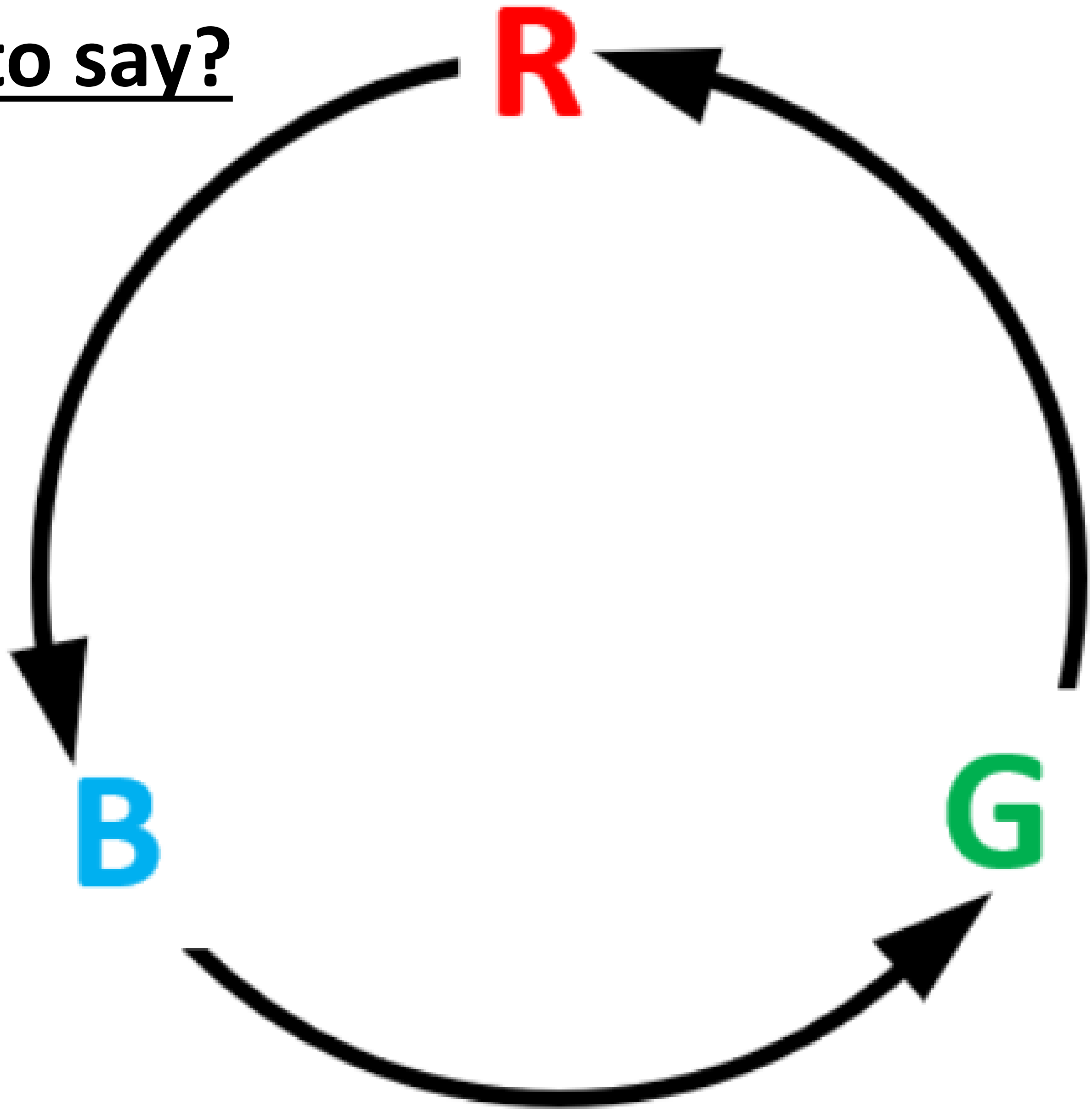
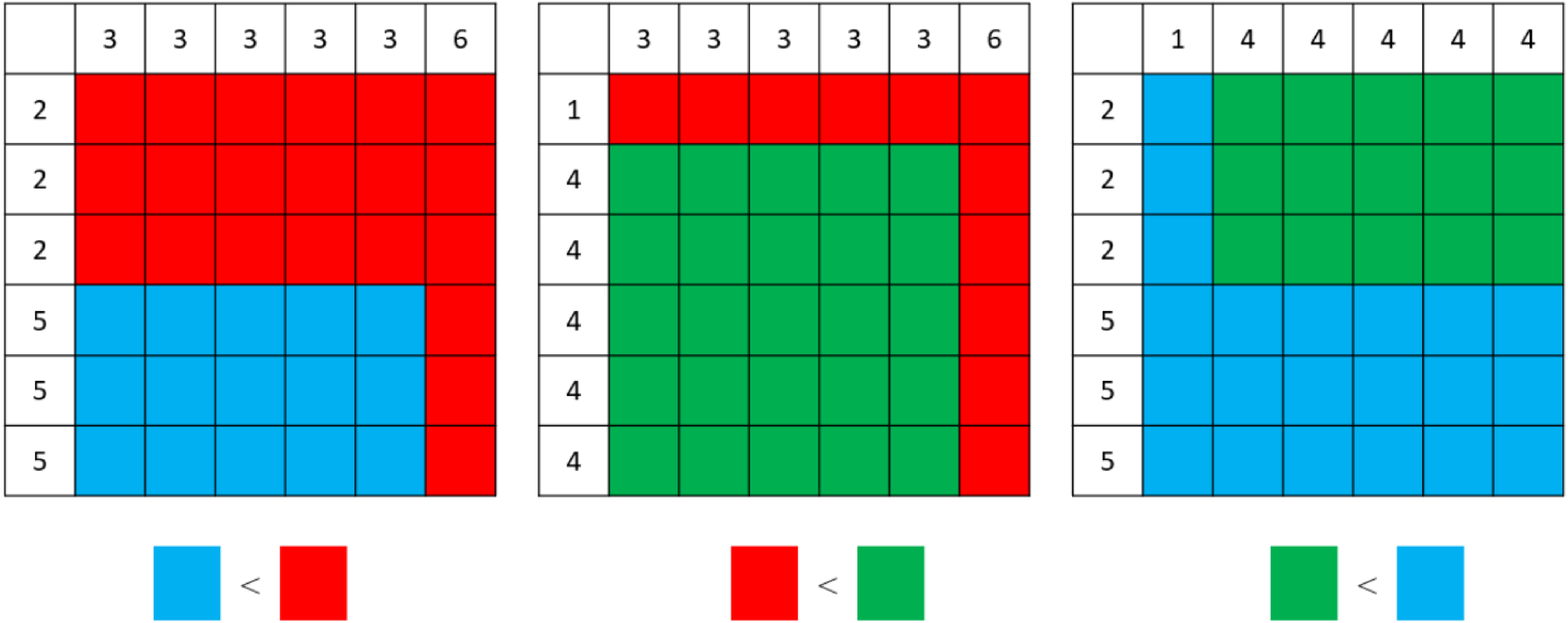
What does math have to say?

All have the same average value, 21/6, so you might think they'll each do equally well.

However, there is a non-transitive ordering, so there is not a "best" die.



We can see this is a diagram: each box is an equally likely outcome, so the total area is the probability of that outcome.



Even crazier, the order reverses with two rolls!

A set of seven dice was found by Oskar van Deventer where for any pair of dice, one can choose a die that beats both on average.

Further questions:

- What about three or four rolls?
- Is van Deventer's the smallest possible set?
- Are there sets for beating any triple or quadruple (what are they)?
- What about dice with other numbers of sides?
- How does a neutral die compare?

	1	1	100
2			
2			
2			

Even if the “average roll” is much higher, a die may still lose more.

This can be made more extreme by adding more sides / increasing 100

This math applies to any random process, like investments or the lottery. The first is like the yellow, it’s more consistent, and the second is like the purple, often worse but occasionally much better. This shows that even if the lottery seems better, you are more likely to do better with stable investments.

More generally, we can assign **probabilities** to each event, the percent chance something will occur, like a roll of a die or one color die beating the other, using the laws of probability.

The probability of both of two **independent** events is the **product** of each event's individual probability.
The probability of one of two **mutually exclusive** events is the **sum** of each event's individual probability.

Independent events are unrelated, one occurring does not affect the probability of the other.
Mutually exclusive events can't both happen at the same time, for example the weather being rainy, or sunny.

	R	5/6	1/6
B		3	6
1/2	2	5/12	1/12
1/2	5	5/12	1/12

	R	5/6	1/6
G		3	6
1/6	1	5/36	1/36
5/6	4	25/36	5/36

	G	1/6	5/6
B		1	4
1/2	2	1/12	5/12
1/2	5	1/12	5/12

The first column / row is the probability, the second is the roll.
The boxes are the probability of each outcome, which are mutually exclusive, so we can add them up to find the probability Of either color die winning.

For example, the probability that red beats blue is 5/12+1/12+1/12=7/12, which is greater than 1/2=6/12, so red is more likely to bead blue than blue is to beat red.

We can now apply these rules to calculate the probabilities of two rolls of each color winning.
We make tables where now the rows / columns are the probability of each possible sum the two dice roll could roll.

	R	25/36	10/36	1/36
B		6	9	12
1/4	4	R	R	R
1/2	7	B	R	R
1/4	10	B	B	R

	R	25/36	10/36	1/36
G		6	9	12
1/36	2	R	R	R
10/36	5	R	R	R
25/36	8	G	R	R

	G	1/36	10/36	25/36
B		2	5	8
1/4	4	B	G	G
1/2	7	B	B	G
1/4	10	B	B	B

- The rolls are independent, so probability multiplies, and the outcomes are mutually exclusive, so we add to find Blue wins with probability

$$\frac{1}{2} \cdot \frac{25}{36} + \frac{1}{4} \cdot \frac{25}{36} + \frac{1}{4} \cdot \frac{10}{36} \approx 0.59 > 1/2$$

Now Blue is more likely to beat Red, as claimed. From the second table, $P(G > R) = \frac{25}{36} \cdot \frac{25}{36} \approx 0.482$, and from the third $P(G > B) = \frac{10}{36} \cdot \frac{1}{4} + \frac{25}{36}\frac{1}{4} + \frac{25}{36}\frac{1}{2} \approx 0.59$