

MATHEMATICAL ART

Brian Mintz
Grad Student Seminar,
Spring 2023

WHAT IS MATHEMATICAL ART?

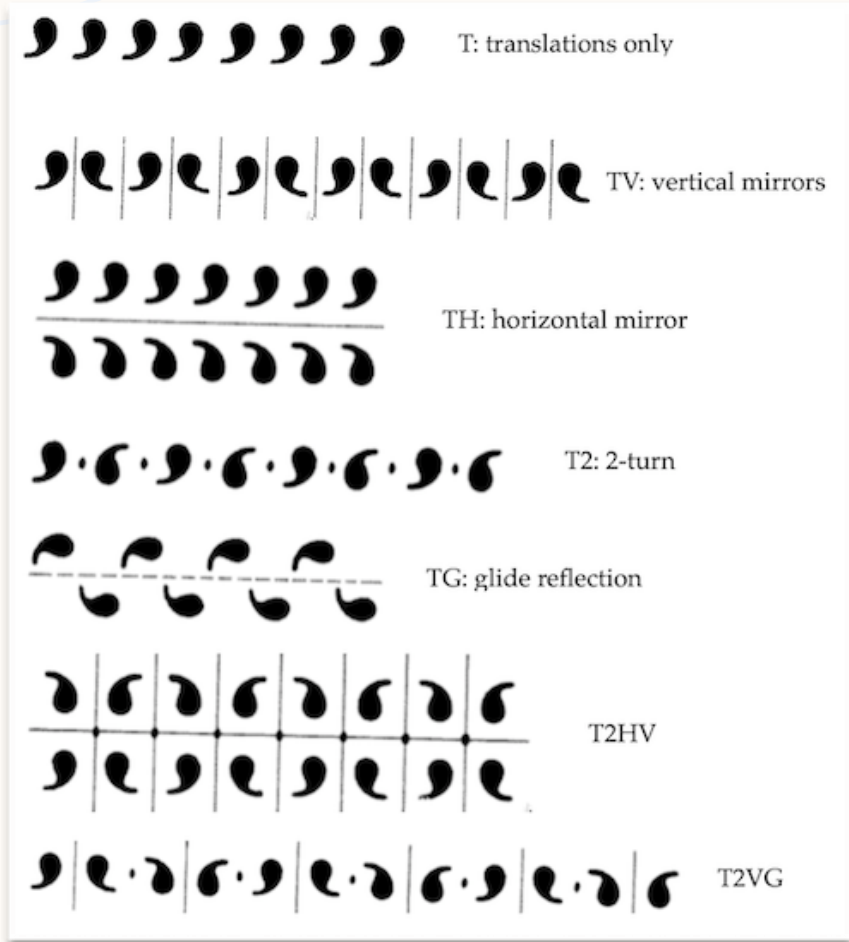
OR, WHY SHOULD I CARE?

Few people get to see the creative side of mathematics. We can share the beauty of patterns and structure in math through traditional art media.

It's also great outreach, an easy answer to "I'm not a math person," or "why would anyone study math?"

This talk will explore some of the myriad ways math has been used in art, and hopefully encourage more people to make and talk about mathematical art!

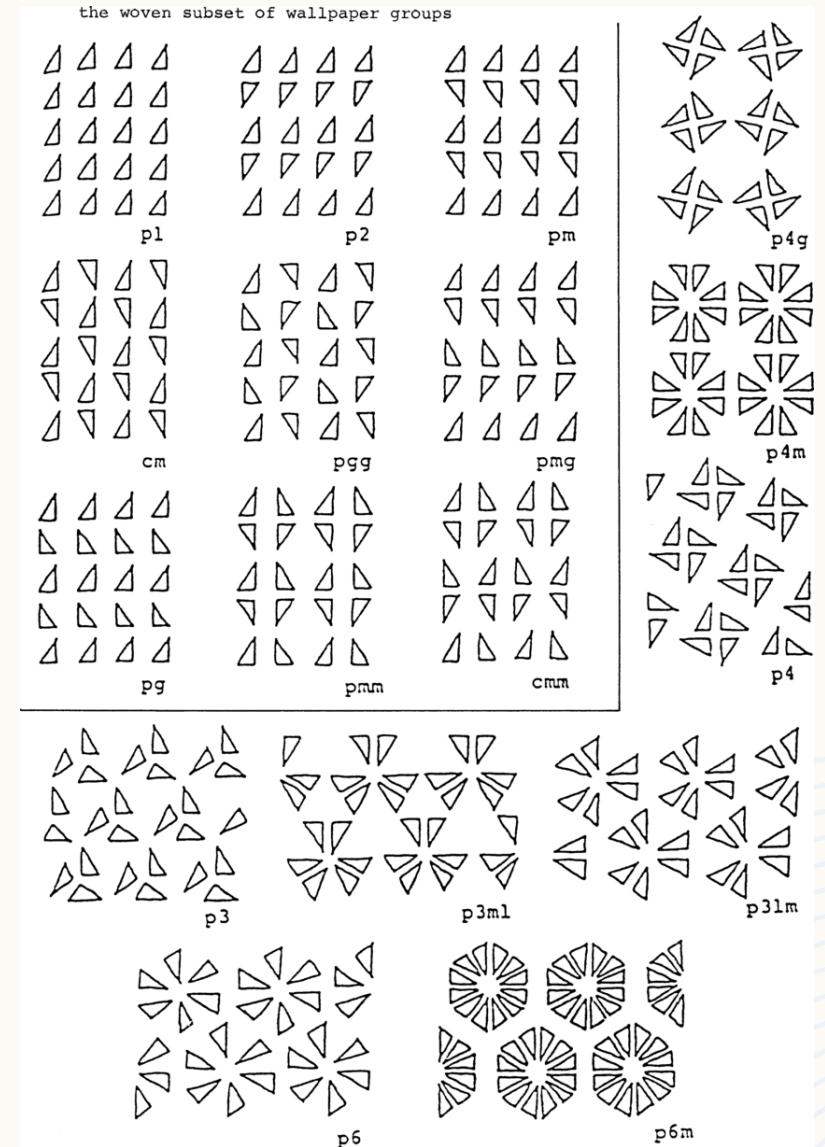
SYMMETRY GROUPS



The symmetry group of a set of points F in R^n is the group of isometries of R^n mapping F onto itself.

This is generated by translations, rotations, reflections, and glide reflections.

Dim	Name	Size
1	Frieze	7
2	Wallpaper	17
3	Crystal / space	230



FRIEZE PATTERNS



Lorelei Koss

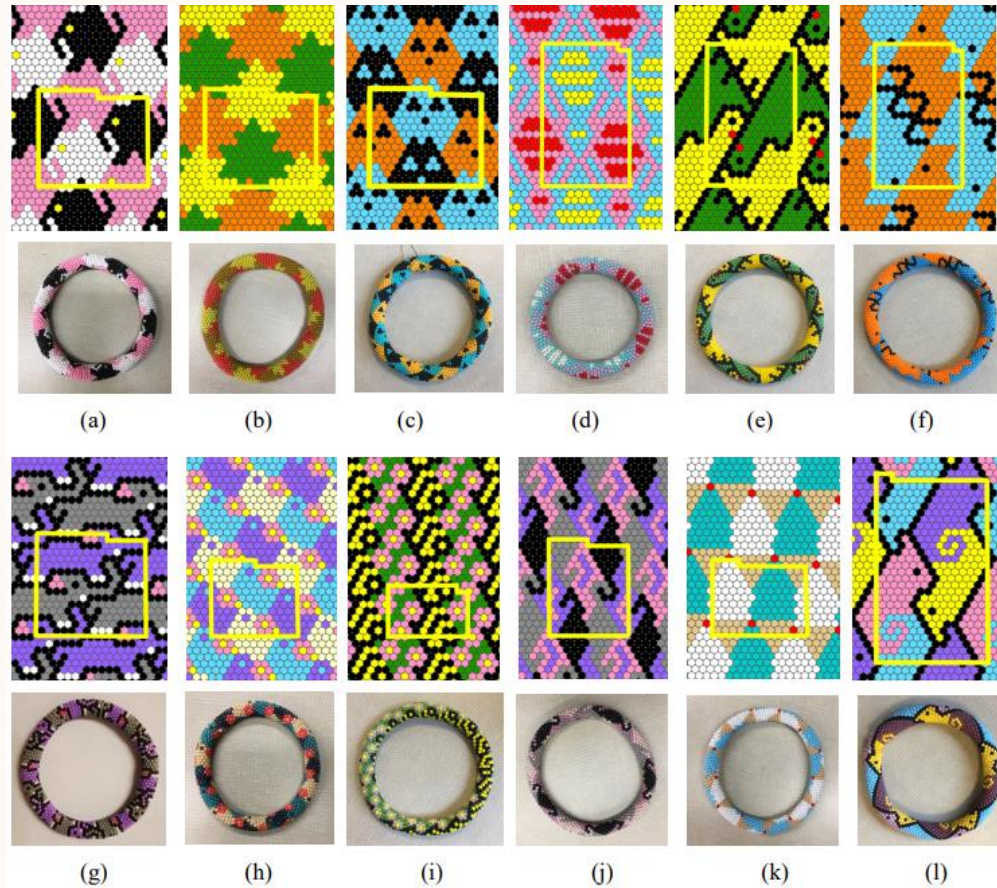
Table 1: Percentages of artifacts found in each frieze symmetry class

Artifacts	p_{mm2}	p_{m11}	p_{ma2}	p_{112}	p_{1m1}	p_{111}	p_{1a1}
European Folk Costumes	37	32	12	5	3	9	2
Lao Textiles	48	20	0	2	19	11	0
Han Textiles	21	23	19	7	3	19	7
Miao Textiles	47	13	26	6	1	4	2
Starkweather Pueblo Pottery	4	13	9	59	2	13	0
Begho Pipes	72	10	4	8	4	2	1
Peranakan Porcelain	1	68	1	10	0	18	1
Pirgí Friezes	41	19	8	11	9	4	9
Ming Porcelain	6	42	8	13	1	18	13
Saudi Arabia Mosques	23	46	1	2	7	17	3
Tonga Handbags	21	42	4	2	2	30	2
Averages from 11 previous studies	29	30	8	11	5	13	4



WALLPAPER SYMMETRIES IN BEADING

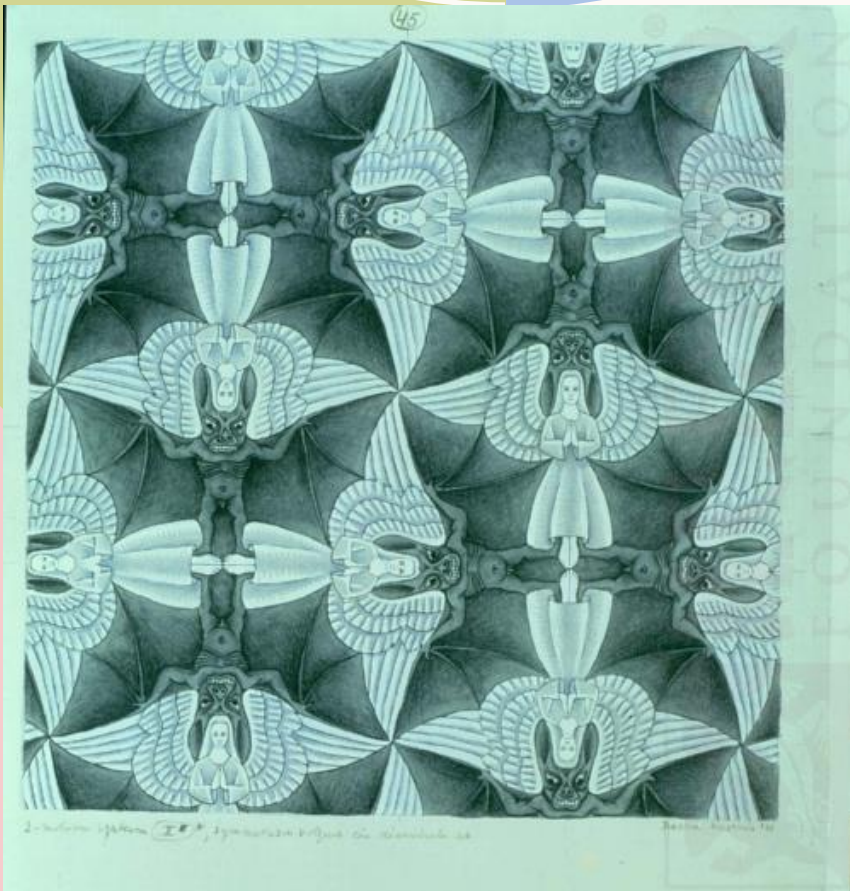
Ellie Baker and Susan Goldstine



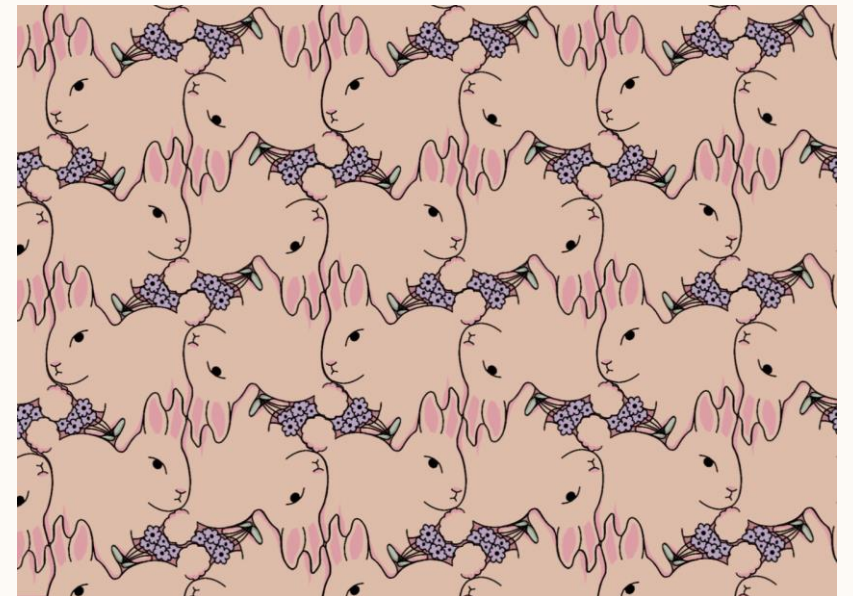
Eve
Torrence

TILINGS

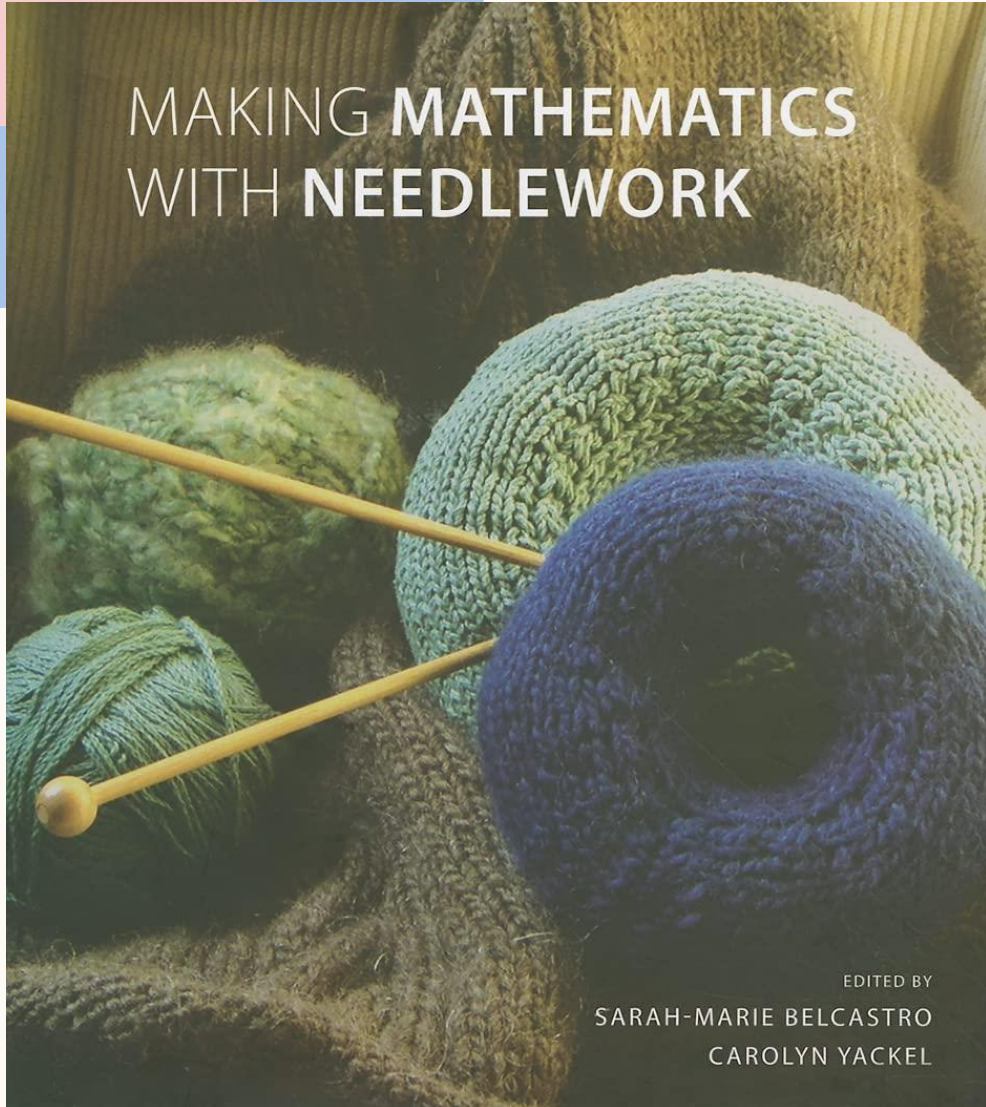
MC Escher



Jiayi Chen, Lucy Knight

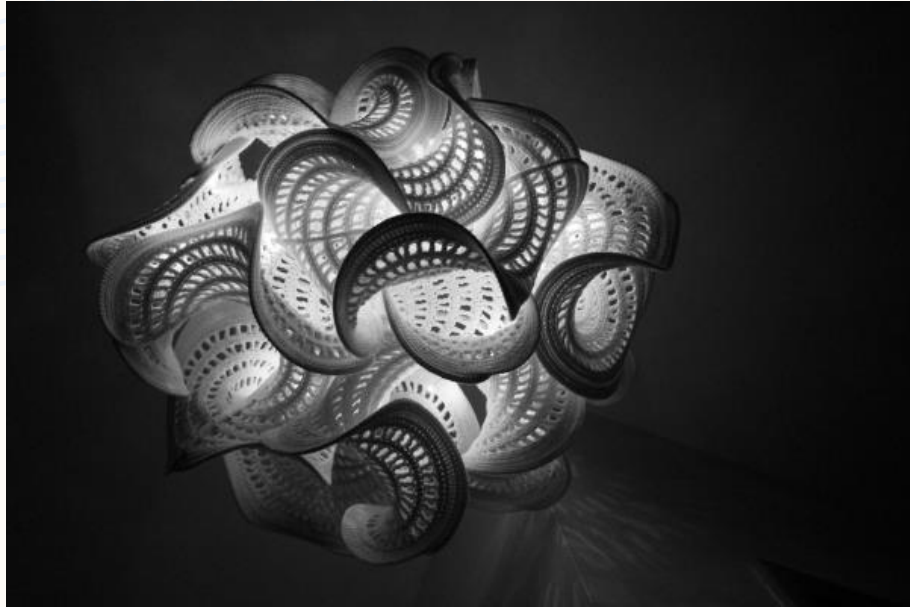


KNITTING



Austin Green





CROCHET

Gabriele Meyer

Shiying dong



Moira Chas



"Blue," by Daina Taimina (Cornell University, Ithaca, NY)

Daina Taimina



ALGORITHMIC ART

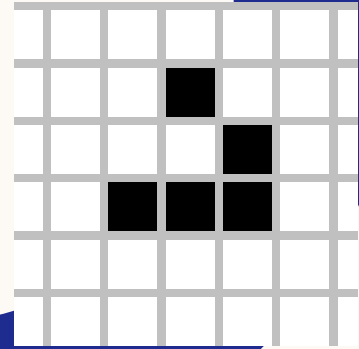
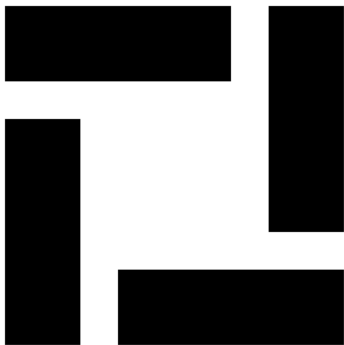
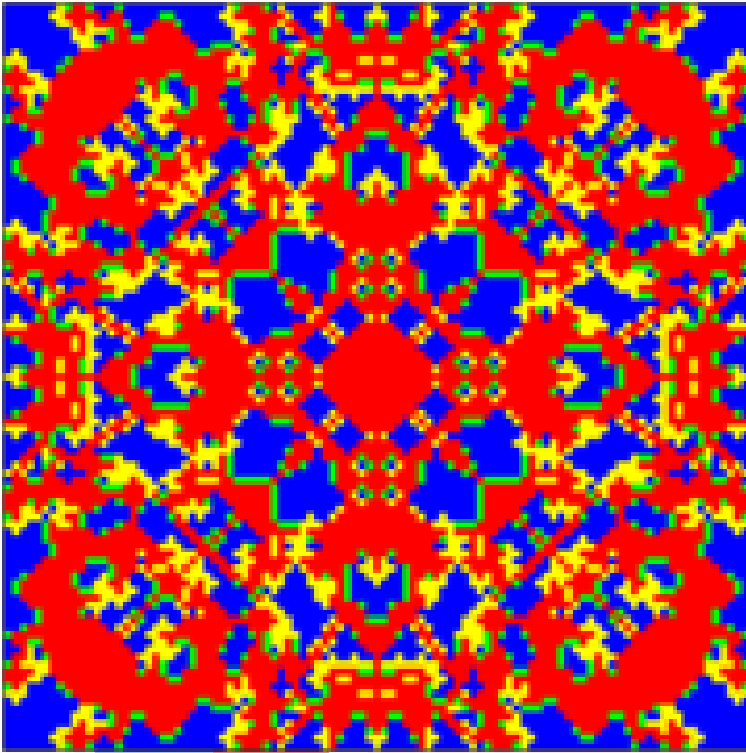
Spatial Games

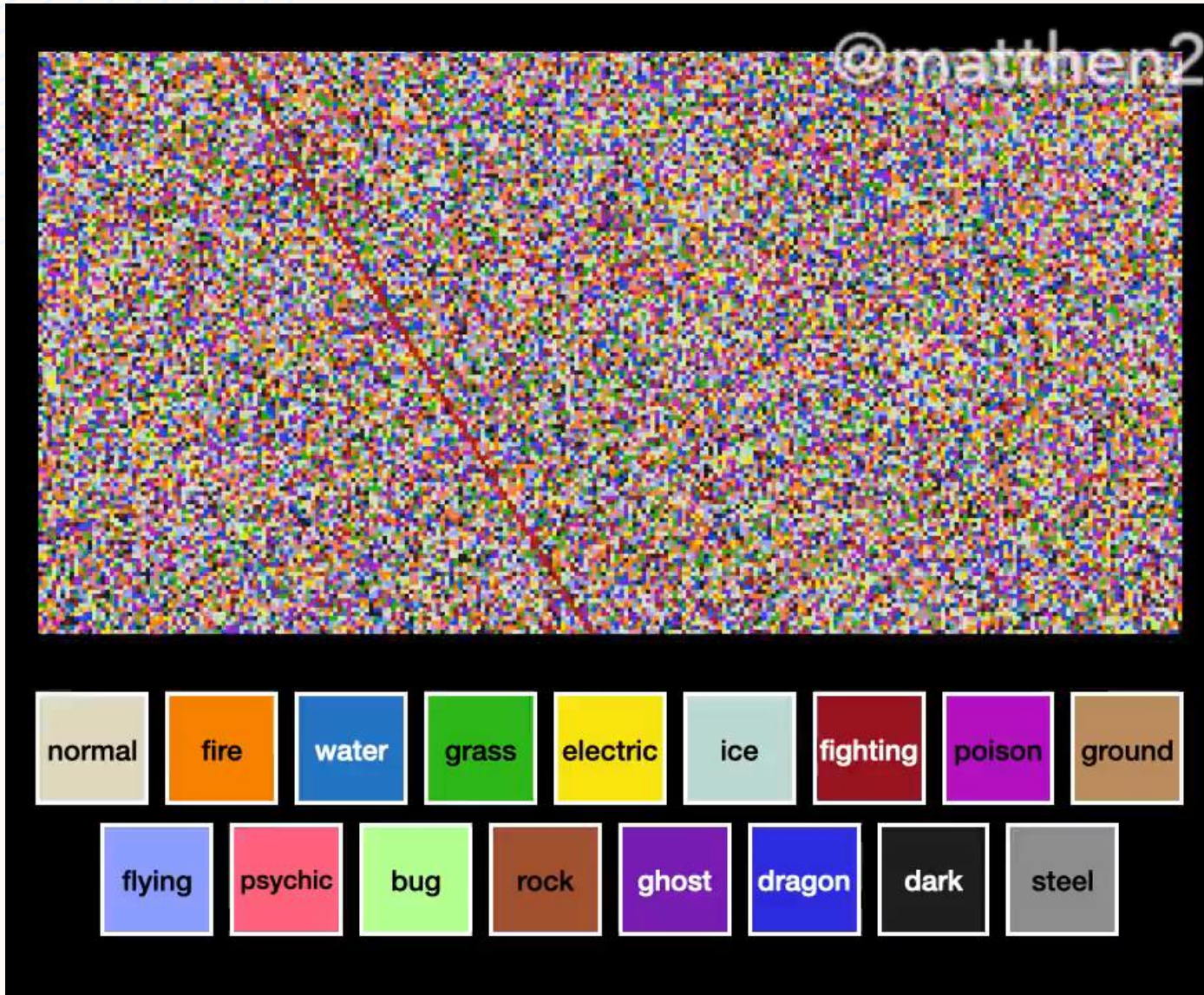
Christoph Hauert

van Dommelen, van Kreveld, and Urhausen

Table 1: *Different systems for figures with a simple procedural definition. The dimension refers to the typical dimension for the image.*

System	Example	Result	Dim.	Properties
Parametrized curve	Lissajoux	curve	2	continuous
	Pendulum	curve	2	continuous
	Spirograph	curve	2	continuous
	Guilloché	curve	2	continuous
	Spirolateral	curve	2	continuous, piecewise-linear
Cellular automata	Game of Life	grid	2	discrete-time
	Sierpiński triangle	grid	2	1D automaton with history
Grammar-based	L-system	shape (object)	2, 3	parallel replacement
	Shape grammar	shape (object)	2, 3	serial or parallel replacement
	Koch snowflake	shape	2	parallel replacement, fractal
Coupled equation system	Hénon map	strange attractor	2	chaotic, discrete-time, quadratic
	Lorenz system	strange attractor	3	chaotic, continuous, quadratic
	Mandelbrot, Julia set	colored plane	2	fractal, iterated function, complex plane
	<i>Spiroplot</i>	<i>point plot, trace</i>	2	<i>discrete-time, linear, multi-point state</i>



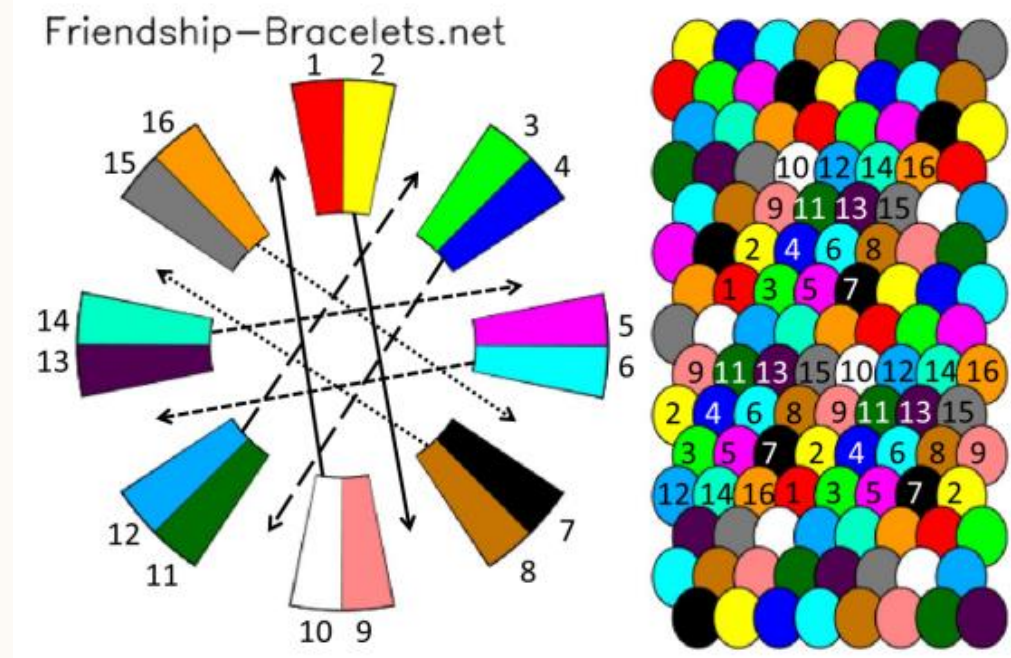


Pokemon spatial game, Matt Henderson



KUMIHIMO

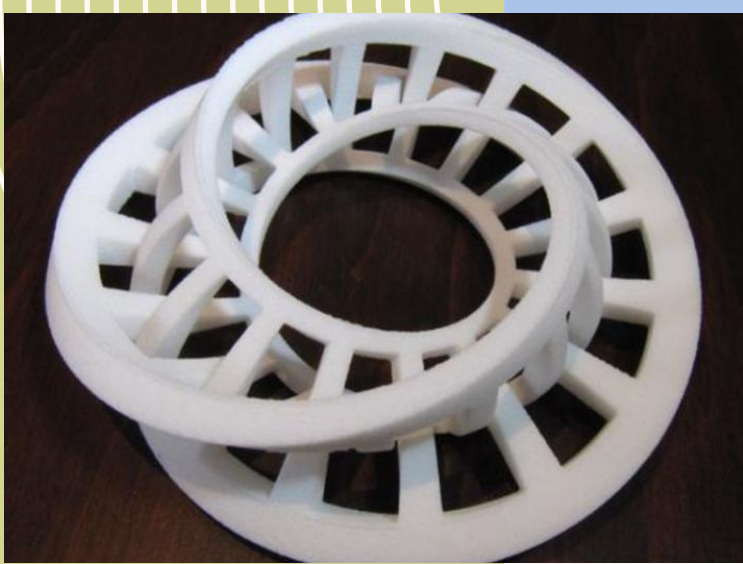
- A Japanese method of braiding.
- Kongō Gumi: 16 strands, 2 colors.
- Joshua Holden counted all symmetric patterns with de Bruijn's generalization of the Pólya enumeration theorem.



Spots	1	2	3	4
#	1	8	21	72
Spots	5	6	7	8
#	147	280	375	257
Total	1161			



Joshua Holden

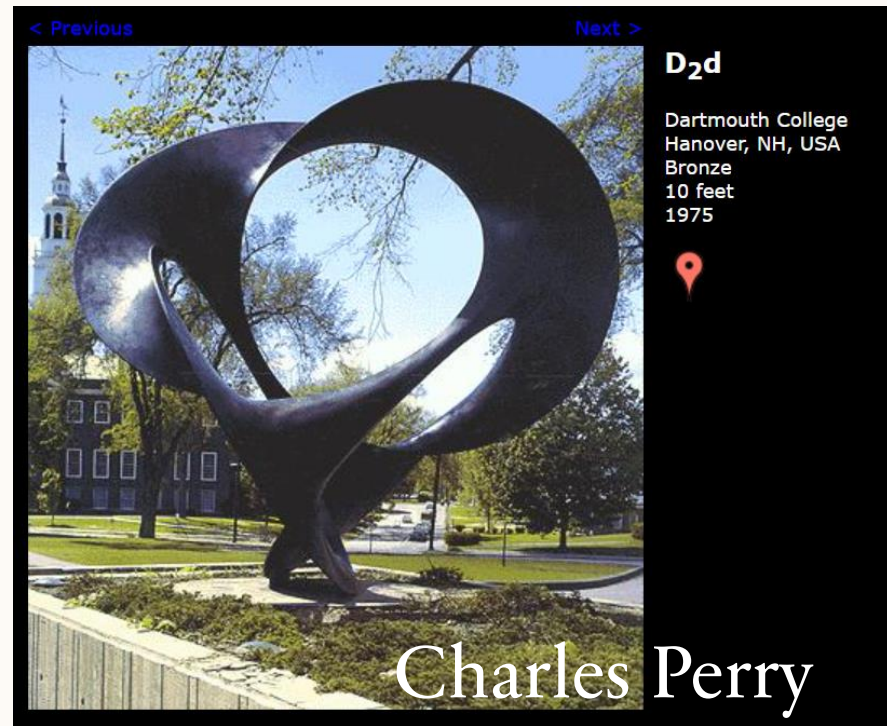


SCULPTURE

Henry Segerman



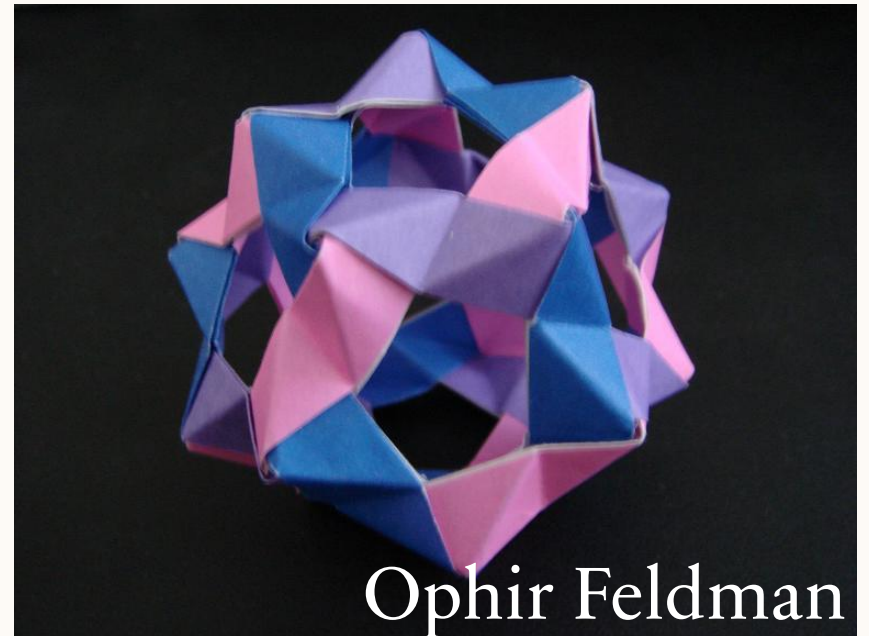
George Hart



D₂d

Dartmouth College
Hanover, NH, USA
Bronze
10 feet
1975

Charles Perry



Ophir Feldman

PUZZLES

Hanayama



Disc

Mobius



Henry Segerman

TEMARI



COOKING



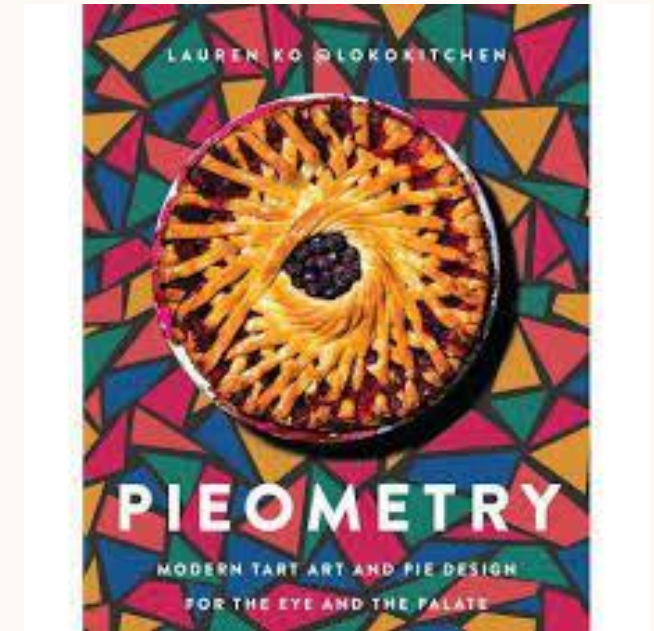
Joy Hsiao

Wafer paper (top left), gum paste (bottom left), dark and white chocolates



Lizzie Buchanan, Beth-Anne Castellano, Brian Mintz, and Alex Wilson.

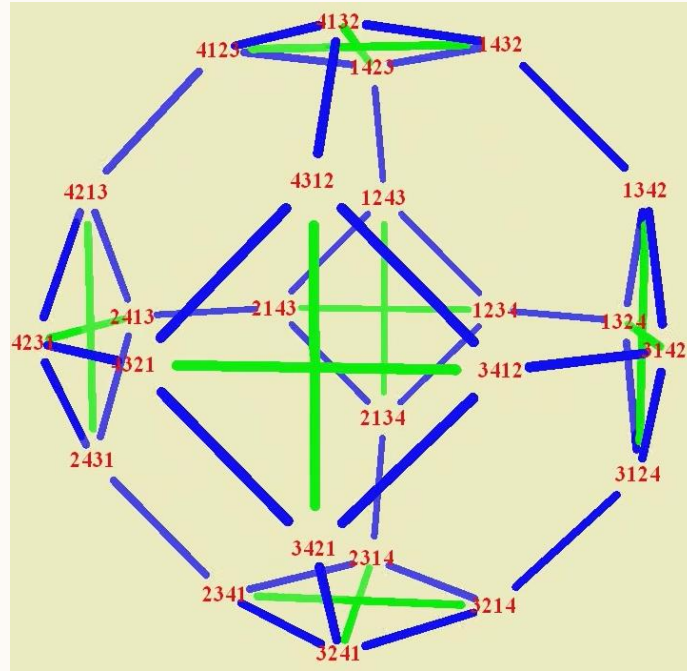
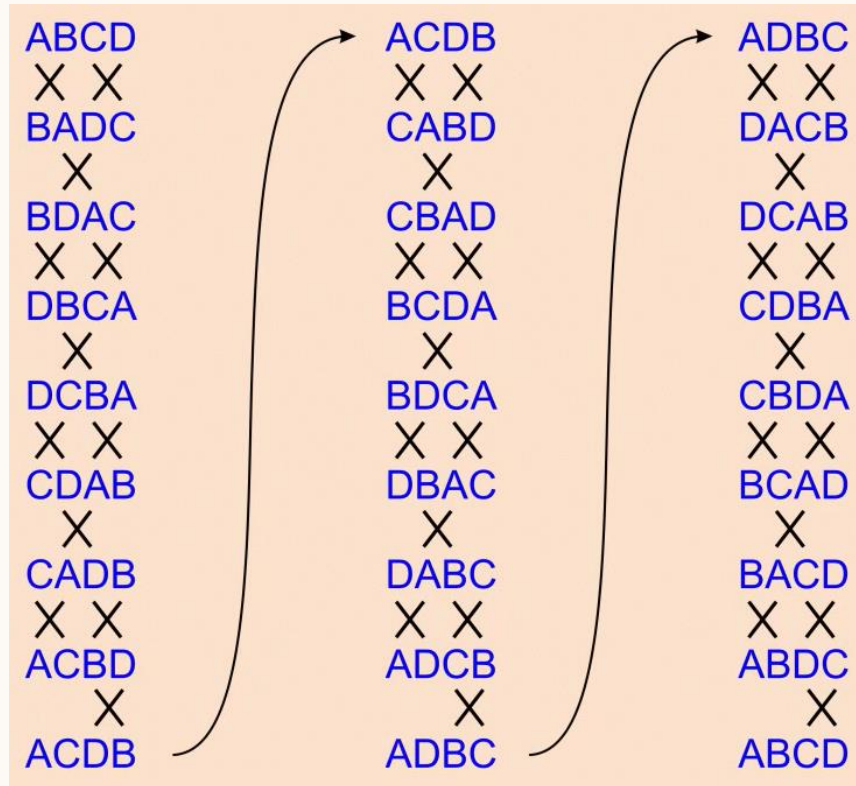
Artic Circle theorem Pie



Cookie Shapes! Vihart

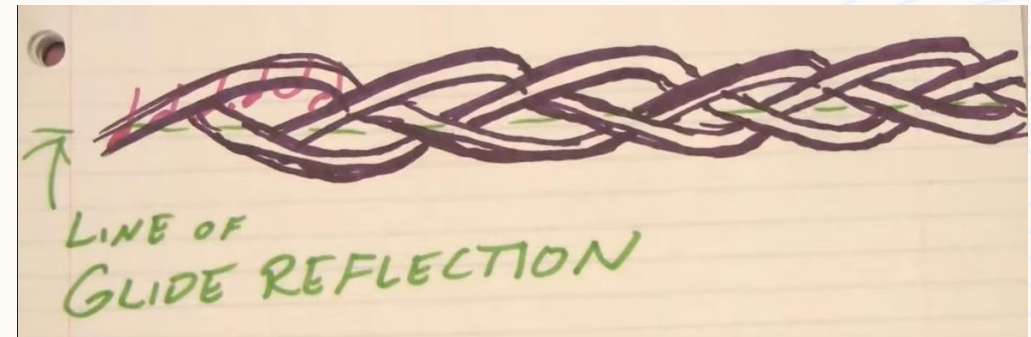
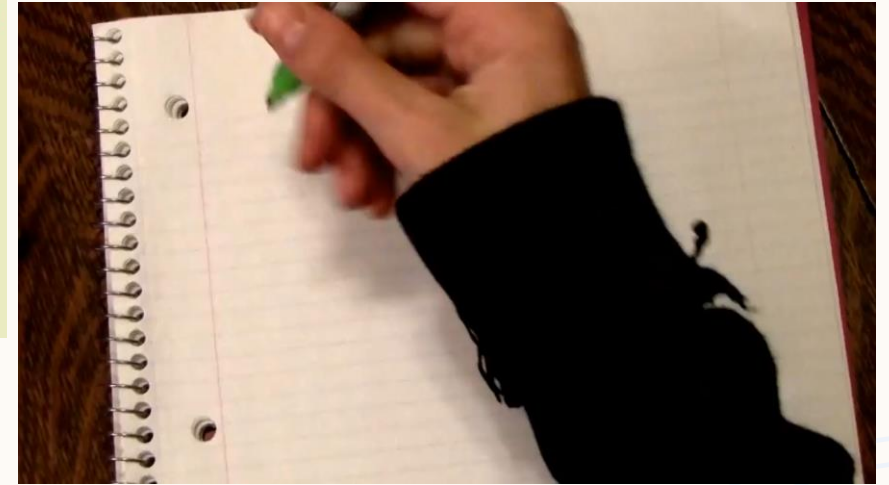


MUSIC



Change Ringing

Sound Braid, Victoria Hart



DANCE

Christine von Renesse



(a) (Simplified) Follower's Movements in Dame (b) (Simplified) Follower's Movements in Dame Dos

Figure 2: Dame and Dame Dos

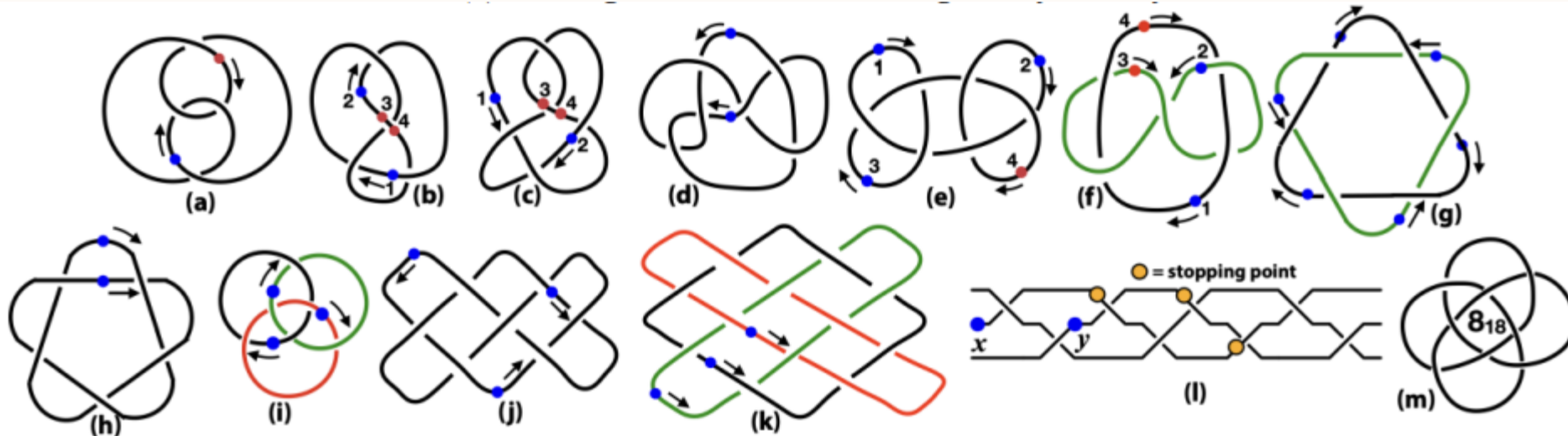
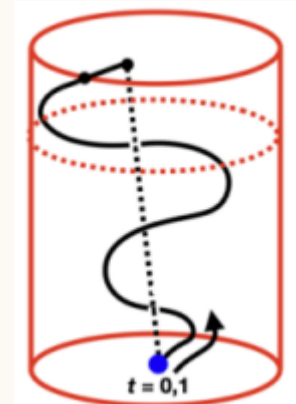


Figure 9: Variety of links and knots and their danceability.

Maypole Karl Schaffer



POETRY

MATH POETRY

Poetry is about as ancient as mathematics in human culture. Its language can be whimsical, somber, joyful, beautiful, concise, thought-provoking, and inspirational. Poems can be limericks, sonnets, haiku, acrostics, square stanzas, based on the Fibonacci sequence (with the number of syllables per line based on the sequence), or other types. Imagine and express mathematics in this unique form!

infinity

is
a color
only artists can
see / yet I see it
in the one one two three five eight
division of Fibonacci / but they burned math at the stake and called her
a heretic / so we ignore the math / we always ignore the math / because /
it can never be as right as the sound a poet makes when he comes / home
after seeing The North Cape by Moonlight for the first time / a computer
could never prove that / I mean this without evidence because to believe
otherwise would be to lose what makes us human / like my mother /
and God / love does not exist / there is no equation for it / math can never
grow / how much I love you / because / zero is the lover / infinity / what
cannot start can never end / the only people who ever die are those who
were born in the first place / infinity / the only people we ever love /
this is what makes us /

—River Oxenreider, The Ohio State University

Sum of Us

See the sum of sequences—of us,
Where each of us is heading.
If any time stopped, our past unlocked,
Without that departure I'm dreading.

See the diverging series—farther,
Farther with every increase.
Our past unlocked if any time stopped,
But come one day, our friendship will cease.

See the converging series—closer,
Closer but never quite there.
If any time stopped, our past unlocked,
With our carefree selves, no tear or wear.

See the sum of sequences—of us,
All we ever amount to.
Our past unlocked if any time stopped,
But one day, I'll truly forget you.

—Angela Zhou, International Academy East

Forest of Numbers

As I walk in the forest it's all so dark,
But as I switched on the light it caused a spark.
Numbers and variables floating away,
Oh I wonder, what shall I learn today?
When I passed 0 the vines all grew,
All the way up to 102.

The sky was grached in ways no one knew,
Clouds were set on points like 0, 0 and 10, 0
Wings of the birds were copied and pasted,
Symmetry all around, no chance wasted.
Aerobically trucks were all so grand,
Rise, Run, and Fall was the name of the band.
The band that played with all the numbers:
Oh, such music caused a shudder.
As all the creatures fell asleep,
Numbers in my head began to divide!
More things, problems, equations to solve
What will I do? I can't do it all!
I call on the forest to help me out,
Oh math, I love math, there is no doubt!

—Miranda Jedlinski, Arthur and Polly Mays Conservatory of the Arts

These poems by middle school, high school, and undergraduate students received AMS Math Poetry Contest awards at the 2023 Joint Mathematics Meetings. Learn about the contest and find resources for exploring math and poetry at www.ams.org/math-poetry.



from: *Quando Che'l Cubo* by Tartaglia

When the cube and the added things
Result in a certain number, find
Two numbers whose difference is this one.

$$[x^3 + ax = b]$$

$$[u - v = b]$$

You should always use these two numbers
Provided their product is equal
Precisely the cube of a third of the thing.

$$[uv = (a/3)^3]$$

Then make it a general rule to subtract the
Cube-roots from each other, in order
To obtain the main thing as your answer.

$$[x = \sqrt[3]{u} - \sqrt[3]{v}]$$

Legato Gelato

Susan Gerofsky

Adepts pasted sateen
The palest pastel petals —
To please those senators asleep to treason

to podiums of the senate,
stapled plates to pleats
whose elapsed duties suited them too well

They tended dented sacred cedars
Dropping a peremptory crusty curtsy
Sirens applied rinses
Ochres thicken
Bruise earth's rubies
Silver livers sliver
She poises her burden
Serves and severs verses,
As the lifter of fares

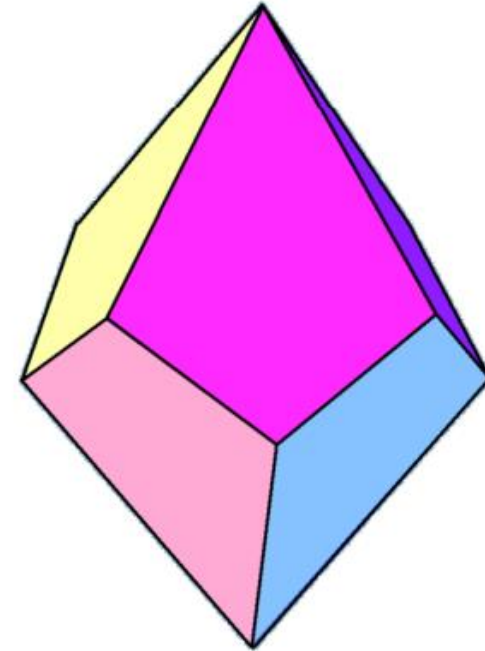
that scared cadres had chopped with chesty scythe
to entrap a curt parent
to resins
kitchen chores
and buries busier hearts
the risen siren
of burned posies
stayed steady
fears and filters safer trifles.

WHY MATH ART? IT'S FUN!

Makoto Nakamura



Andrea Hawksley



Tetragonal Trapezohedron ©Tomruen, CC BY-SA 4.0

Figure 3: *The otterhedron (left) is made from 8 stuffed otters arranged like a tetragonal trapezohedron (right).*

MATH CAN BE ADDED TO ANYTHING, WHAT COULD YOU MAKE?

THANK YOU!

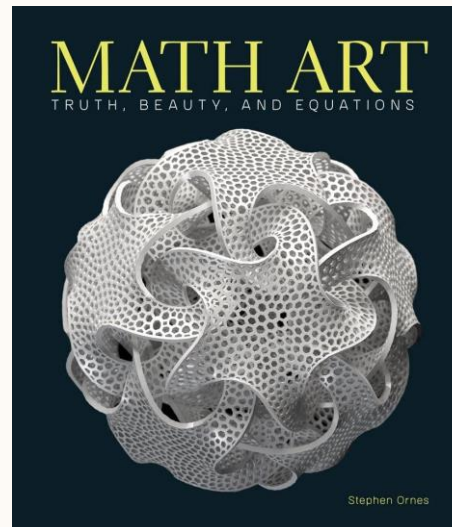
This presentation is available on my website if you'd like to follow up with any of these:



I'm also happy to chat more!

A FEW REFERENCES

- Journal of Mathematics and the Arts
- <http://www.toroidalsnark.net/mathknit.html>
- <http://woollythoughts.com/>
- <https://www.artofmathematics.org/>
- <https://mathcraft.wonderhowto.com/>



AND MUCH MORE...

<https://www.mathed.page/geometry-labs/symmetry-labs.html>

https://www.researchgate.net/figure/The-17-wallpaper-groups-with-their-simple-patterns-all-drawings-of-paper-are-by-the_fig1_228781255

<https://archive.bridgesmathart.org/2021/bridges2021-253.pdf>

<https://nrich.maths.org/1341>

https://www.maa.org/sites/default/files/images/upload_library/4/vol1/architecture/Math/seven.html

<http://faculty.smcm.edu/sgoldstine/gallery/CBS.html>

<https://archive.bridgesmathart.org/2021/bridges2021-1.pdf>

<https://mcescher.com/gallery/symmetry/>

<https://canvas.dartmouth.edu/courses/56422/pages/esc-her-gallery>

<https://www.americanscientist.org/article/adventures-in-mathematical-knitting>

http://gallery.bridgesmathart.org/exhibitions/2020-joint-mathematics-meetings/gabriele_meyer

<http://www.ams.org/publicoutreach/math-imagery/taimina>

<http://www.ams.org/publicoutreach/feature-column/fc-2018-05>

<http://faculty.smcm.edu/sgoldstine/torus7.html>

<http://wiki.evoludo.org/index.php?title=EvoLudo>

<https://archive.bridgesmathart.org/2020/bridges2020-353.pdf>

https://commons.wikimedia.org/wiki/Category:Animations_of_the_Game_of_Life

<https://twitter.com/juanbuis/status/1600155605112496129>

<https://twitter.com/matthen2/status/1543226572592783362>

<https://archive.bridgesmathart.org/2022/bridges2022-327.pdf>

<https://www.shapeways.com/product/VULEQP5GU/interlocking-mobius-ladders?optionId=42989409&li=shops>

<https://www.georgehart.com/sculpture/frabjous.html>

<https://www.maa.org/programs/faculty-and-departments/curriculum-department-guidelines-recommendations/innovative-teaching-exchange/oragami-geometry-classroom>

<https://www.youtube.com/watch?v=Cp3q6NduvOU>

<https://japanobjects.com/features/temari>

<https://temaritwins.com/wp-content/uploads/2018/12/Karens12puzzlingshapessmall.png>

<https://www.flickr.com/photos/barbarabsuess/51368969625/in/album-72157719697510000/>

https://www.reddit.com/r/temari/comments/o4kup4/temari_2_259_cm_circumference/

<https://mymodernmet.com/temari-balls-geometric-patterns/>

<http://gallery.bridgesmathart.org/exhibitions/2021-bridges-conference/joy-Hsiao>

https://www.youtube.com/watch?v=_n1126GoxbU

<https://www.youtube.com/watch?v=VB6a4nI0BPA>

<https://www.youtube.com/watch?v=3lyDCUKsWZs>

<https://archive.bridgesmathart.org/2021/bridges2021-79.pdf>

<https://www.familyhandyman.com/article/what-is-a-maypole-and-why-do-people-dance-around-it/>

<https://archive.bridgesmathart.org/2020/bridges2020-513.pdf>

<http://www.ams.org/programs/students/math-poetry>

<https://archive.bridgesmathart.org/2020/bridges2020-273.pdf>

<https://archive.bridgesmathart.org/2022/bridges2022-79.pdf>

<https://archive.bridgesmathart.org/2022/bridges2022-359.pdf>

<http://gallery.bridgesmathart.org/exhibitions/2014-bridges-conference/makoto-nakamura>