# Hackenbush

This is a two player game invented by John Conway with some cool math behind the scenes!

**How to play**:

- A game starts with a collection of edges colored red and blue, at least one connected to the base line.
- Each player chooses a color, then starting with red, players alternate removing a single edge of their color from the picture (by scribbling over it).
- Any edges that become disconnected from the base line are also removed.
- The last player to make a move wins (this happens when there are no edges of the other player's color).

**Example**:



No moves for red, so blue wins
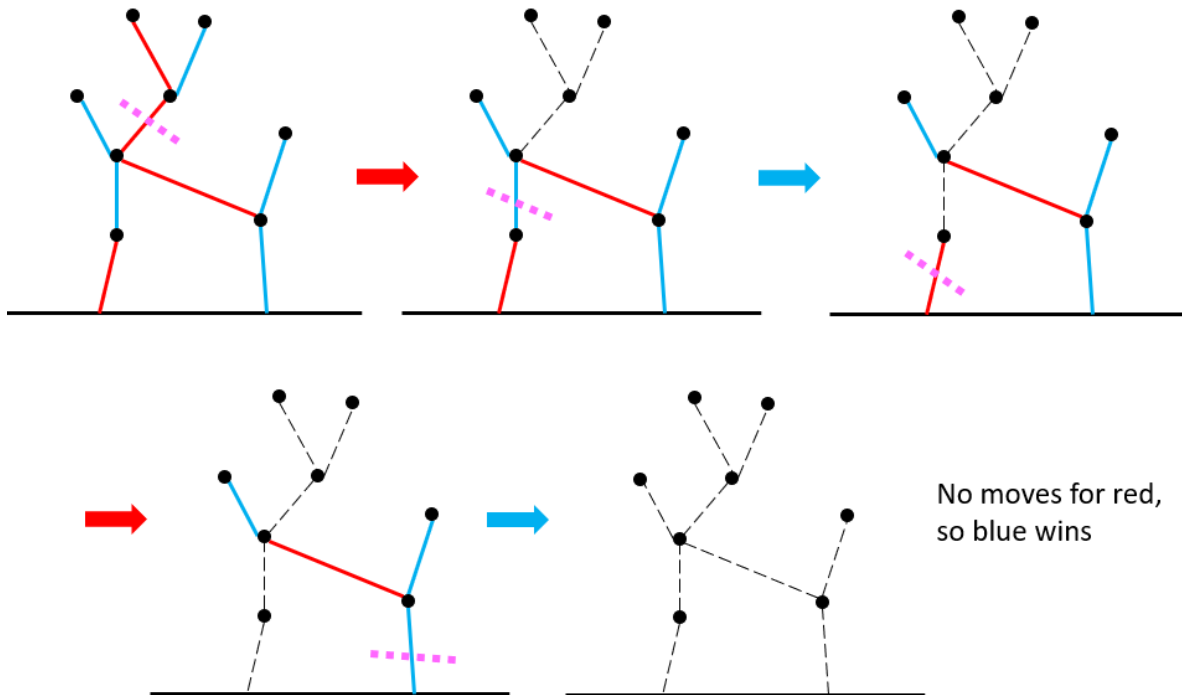
**Try it yourself!**
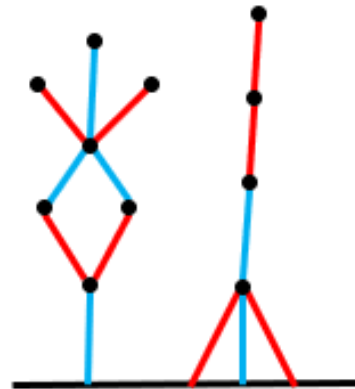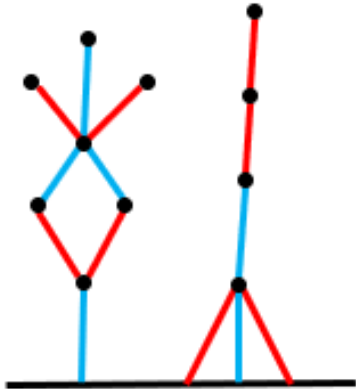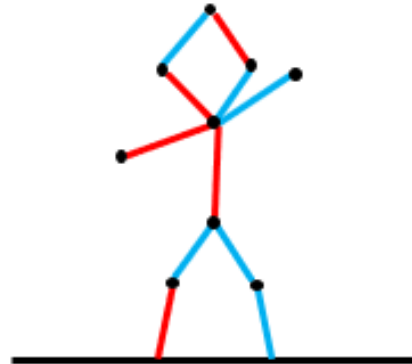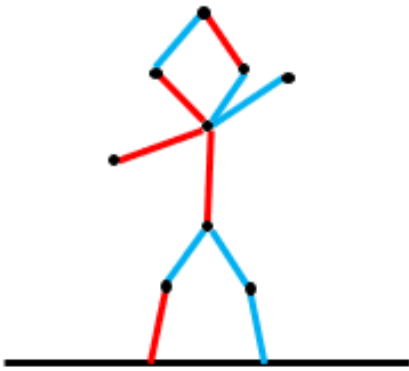


## Make your own!

**Questions**:

- Which games are easier for red or blue to win?
- How do you decide which edges to remove?

# The Math

You probably noticed some games are easier for one color to win. We can quantify this by giving each game a score, which represents how many extra turns red would need to win. If this score is positive, blue will win if they play perfectly; if it is negative, red will win if they play perfectly.

Try to make moves that yield games with a large positive score if you're the blue player, or a large negative score if you're the red player.

It turns out that the score we give a game is the sum of each separate picture that makes it up. In addition, switching the colors of all edges multiplies the score by minus one.

**Example**: (using the values on the back)

$$\frac{1}{4} \quad -3 \quad + \quad 1 \quad = -1\frac{3}{4} \qquad \frac{3}{4} \quad -\frac{1}{2} \quad = \frac{1}{4}$$
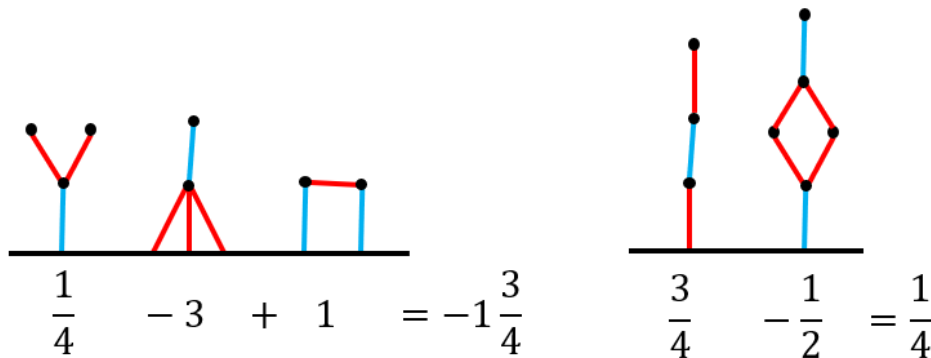
The scores of these games can be thought of as 'Surreal numbers,' which include not only every real number but also infinitely large and small numbers. We already know how to add and subtract games (and thus surreal numbers), but games can also be multiplied and divided!

**References**: You can learn more at the references under the Hackenbush section on the following website: https://math.dartmouth.edu/~bmintz/other.html

**Reference table**:
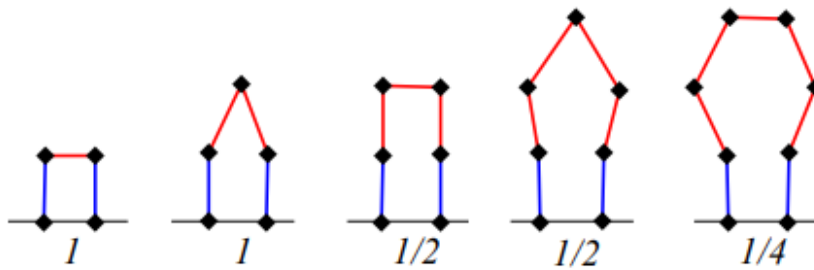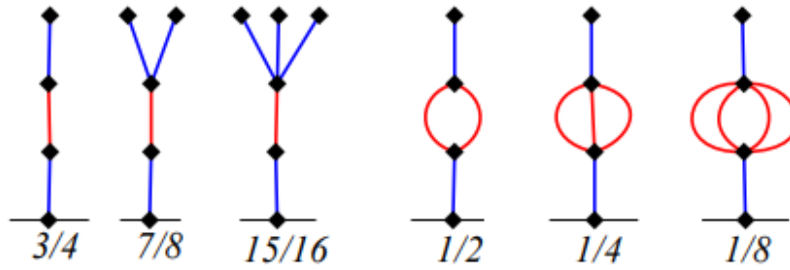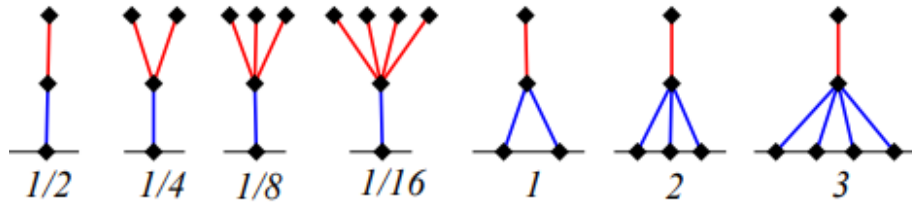


See if you can use this table to win these games!