The Evolution of Pro-Social Norms

Compassion, Universalizability, Reciprocity, and Equity, a C.U.R.E for social dilemmas

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Introduction

A longstanding question in biology is where cooperation comes from. Several organisms display remarkable behaviors that depend on intricate coordination.

But how does this come about through natural selection?

Prior work

Martin Nowack[1] identified five methods for evolution to select for cooperation:

- kin selection,
- direct reciprocity,
- indirect reciprocity,
- network reciprocity,
- group selection.

Other strategies have been shown to be able to invade defecting populations.[2]

[1] "Five rules for the evolution of cooperation" by Martin A. Nowak

[2] "Evolutionary instability of selfish learning in repeated games" by Alex McAvoy, Julian Kates-Harbeck, Krishnendu Chatterjee and Christian Hilbe.

Compassion in spatial games

Few people solely try to maximize their own payoff, most consider how their actions will effect others (even if only a little bit).

Szabo et al.[3] studied a spatial prisoner's dilemma and Hawk-Dove games where individuals chose their strategy x to maximize $(1-Q)p(x,y)+Qp(y,x)$.

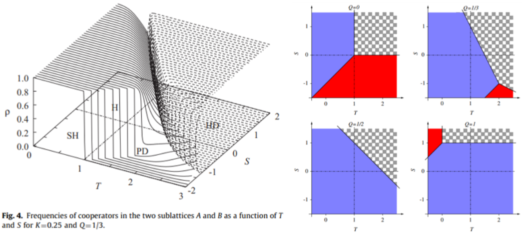

Fig. 4. Frequencies of cooperators in the two sublattices A and B as a function of T and S for $K=0.25$ and $Q=1/3$.

[3] "Selfishness, fraternity, and other-regarding preference in spatial evolutionary games" by G Szabo, A Szolnoki

Model

Each player has a genetically determined value v and a default strategy x.

Individuals interact uniformly at random, reproducing based on the payoff the receive.

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Based on their value v and the other player's default strategy y, players act according to the strategy x^* that maximizes their utility, a linear combination of two goals: payoff and social responsibility:

$$x^* = \mathrm{argmax}_x \, (1 - v)p(x, y) + vf(x, y)$$

Where $p(x, y)$ is the payoff of strategy x when interacting with strategy y, and $f(x, y)$ is some function encoding a social norm.

Key slide 1: Possible norms

norm	$f(x)$
compassion	$p(y, x)$
univeralizability	$p(x, x)$
reciprocity	$\exp(-(x-y)^2)$
equity	$\exp(-(p(x,y)-p(y,x))^2)$

Note there are several other functions that would work. For example, if $z = p(x,y) - p(y,x)$ is the payoff difference, equity (fairness) could be described by $-|z|$, $-z^{2n}$, or $\frac{1}{z}$.

Symmetric Two Player, Two Action Games

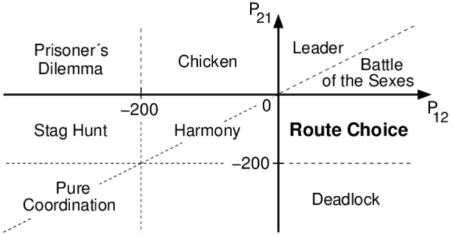
Two players each choose from two actions P or Q, determining the payoffs each receives according to the same table.

Payoffs	P	Q
P	a	b
Q	c	d

$\sim$

	0	x
	y	1

Rescaling and permuting gives a two dimensional space of games.



From "How Individuals Learn to Take Turns: Emergence of Alternating Cooperation in a Congestion Game and the Prisoner's Dilemma" by D. Helbing and M. Schönof

Prisoner's Dilemma

In this work, we'll focus on the prisoner's dilemma, since this is a canonical case where cooperation is selected against.

	C	D
C	R	S
D	T	P

In this game, $S < P < R < T$, so we'll use a different normalization to represent all possible games in a bounded region: $S = 0$ and $T = 1$.

Strategies are probability distributions $xC + (1 - x)D$, with payoffs the expected value

$$p(x, y) = Rxy + Sx(1 - y) + T(1 - x)y + P(1 - x)(1 - y)$$

Population fitness

The function $p(x, x) = Rx^2 + (S + T)x(1 - x) + P(1 - x)^2$ gives the payoff received when all players follow the same strategy, so represents the fitness of a strategy adopted throughout the population.

Note this may look differently even withing a class of games, e.g. the below games are both prisoner's dilemmas, yet the first makes $p(x, x)$ concave up while the other is concave down.

	C	D
C	0.2	0
D	1	0.1

	C	D
C	0.9	0
D	1	0.8

This means that even though mutual cooperation is the best outcome individually, on the population level it is possible for some defection to be a good thing.

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Here we take $f(x, y) = p(y, x)$, so individuals care about the other player getting a good payoff too.

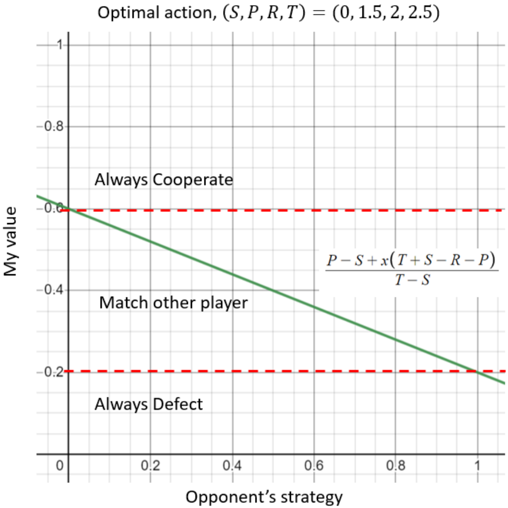
Since both $p(x, y)$ and $f(x, y)$ are linear, the optimal action will be a pure strategy, zero or one. Which it is depends on if c is greater than the threshold satisfying

$$(1 - c_y)p(0, y) + c_y p(y, 0) = (1 - c_y)p(1, y) + c_y p(1, y)$$

This is solved by

$$c_y = \frac{P - S}{T - S} + y\frac{(T + S - R - P)}{T - S}$$

Key slide 2


Optimal action, $(S, P, R, T) = (0, 1.5, 2, 2.5)$
My value
Opponent's strategy
1
0.8
0.6
0.4
0.2
0
0.2
0.4
0.6
0.8
1
Always Cooperate
Match other player
Always Defect
$\frac{P - S + x(T + S - R - P)}{T - S}$

An exception

Note if the coefficient $\frac{(T+S-R-P)}{T-S}$ of y is positive, in this case $T+S-R-P>0$, this interpretation doesn't work.

There are still individuals who always cooperate and defect, above and below the intersections with $y=0$ and $y=1$, but the intermediate players now choose the **opposite** strategy of the other player. That is, the cooperate with defectors and defect with cooperators.

Consequently, they will invade cooperating populations, and die out in defecting ones, causing the value to decrease over time.

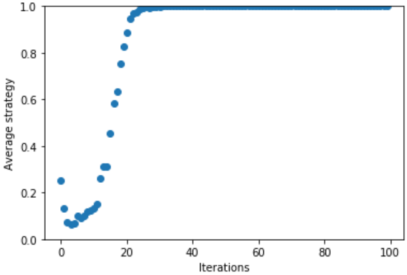
Agent Based Model: Fisher Process

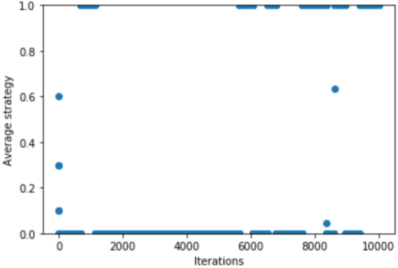
We have a finite population of n individuals.

Each round:

- Each individual interacts with a random other player to determine its fitness.
- n individuals are selected at random, proportional to their fitness and with replacement, for the new population (potentially with some mutation).

Simulating a population near the threshold value, we see how cooperation can take over.





Further, allowing rare mutations in the value results in the population alternating between defecting and cooperating states.

Stochastic Model: Moran Process

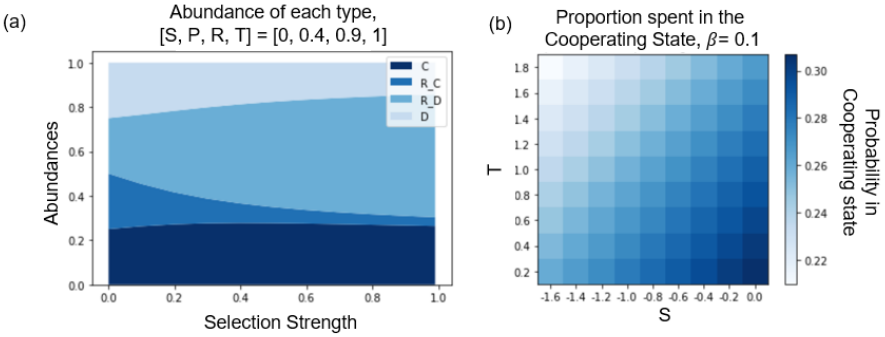
We saw there are effectively four types of players, depending on their value:

- Cooperators - always cooperate.
- Reciprocating (currently playing C or D) - imitate the other players strategy
- Defectors - always defect.

This makes the continuous trait space into a discrete one, which is more tractable (however we've lost information about the value, so can't include mutation).

So we have a Moran process with a finite population: at each step a pair is chosen uniformly at random, and one replaces the other with probability proportional to their fitness. States are partitions of n into four parts: (C, R_C, R_D, D).

Taking the limit of low mutation, the population is approximately monomorphic. One can calculate the fixation probabilities of any mutant, given the selection strength β, making the process a Markov chain over monomorphic states. It's stationary distribution, given by the principle eigenvector, says the time the population stays in each state.

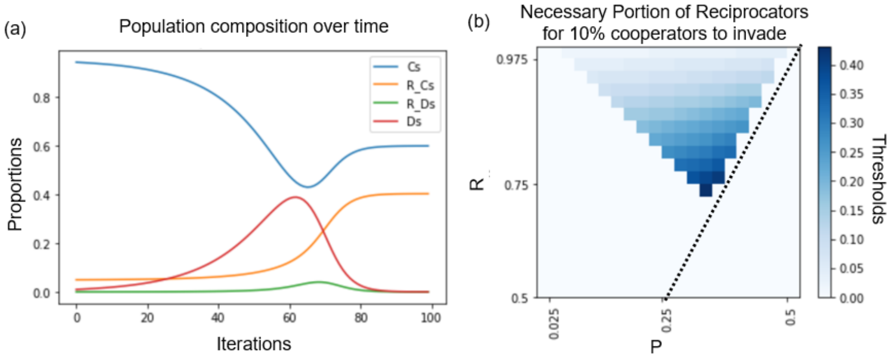


Deterministic Model: Large n limit

Taking the limit as the population size goes to infinity, there are enough individuals for all possible interacting pairs to occur proportionally to their relative frequencies. This gives the proportions of each type at the next step as:

$$
\begin{aligned}
C' &= (R(C + R_C + R_D) + SD)C \\
R'_C &= R((R_C + R_D)C + R_C R_C) + SR_D R_C \\
R'_D &= P((R_C + R_D)D + R_D R_D) + TR_C R_D \\
D' &= (TC + P(R_C + R_D + D))D
\end{aligned}
$$

Invasion Dynamics



You can play around with the game parameters and initial proportion of reciprocating players to explore these dynamics using a Python Notebook hosted on Google Colab at my website.

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Universalizability

Now $f(x, y) = p(x, x)$ is a quadratic function, leading to two distinct cases.

If $f(x, y)$ is concave up, the optimal strategies are only zero or one, so very similar analysis as in the compassion case can be used. Now the threshold value satisfies

$$\begin{aligned}(1-u_y)p(0,y)+u_y p(0,0) &= (1-u_y)p(1,y)+u_y p(1,1)\\ \rightarrow u_y &= \frac{P-S-y(R-S-T+P)}{R-S-y(R-S-T+P)}\end{aligned}$$

and we again have our four types C, R_C, R_D, and D.

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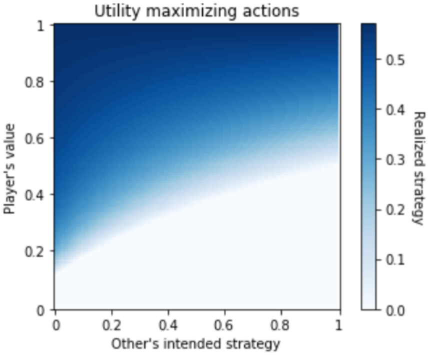
However, if $f(x, y)$ is concave down, the optimal strategy can be any intermediate value. This occurs in the same region $P + R < S + T$ that was pathological for the compassion norm.

Concave down case

Simulation showed larger values could not invade. This makes some geometric sense, by thinking of u as interpolating between $p(x, y)$ and $p(x, x)$.



Increasing u shifts the realized strategy up, but not enough others will cooperate more.

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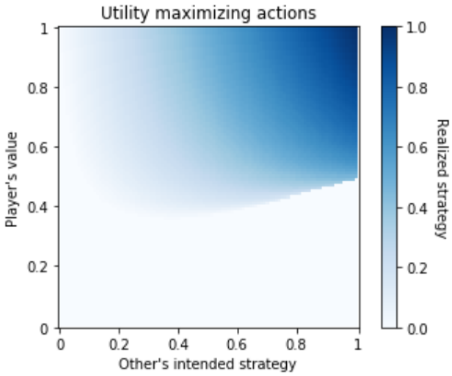
Reciprocity

Here $f(x, y) = \exp(-(x - y)^2)$, and a little algebra shows the equity norm $f(x, y) = \exp(-(p(x, y) - p(y, x))^2)$ is a multiple of this for this game, so qualitatively similar.



Simulation results show this can't lead to cooperation.

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Main Results

- We introduce a model that quantifies adherence to a pro-social norm that is general enough to encompass a wide range of possible norms.
- Our model is continuous, allowing for smooth transitions between cooperation and defection.
- Through Agent-based simulation and analysis of stochastic and deterministic models, we characterize when a range of norms can promote cooperation.
- The compassion and universalizability norms allow for cooperation through reciprocating types with intermediate values. Further, the give remarkably similar results, indicating a potential generality.
- However, Reciprocity and Equity were unable to promote cooperation on their own.

Next Steps

- Consider evolution in the norm itself, perhaps in one of its parameters, or with one value for each norm, or two well mixed population interacting with at some rate.
- Explore different games, e.g. Hawk-dove has less intense competition, or public goods games / spatial games.
- Identify general criteria necessary for a norm to promote cooperation.

Link to my website with slides and more!